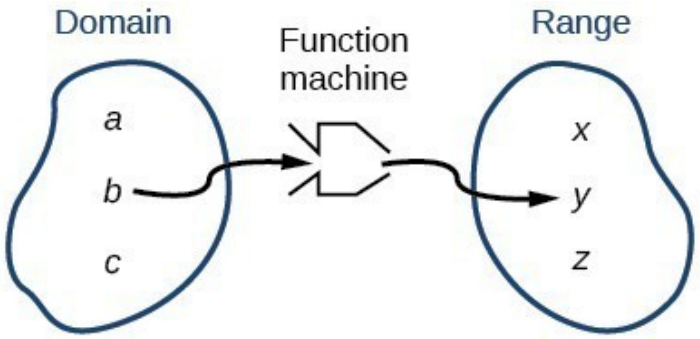
1.1 Functions

Function

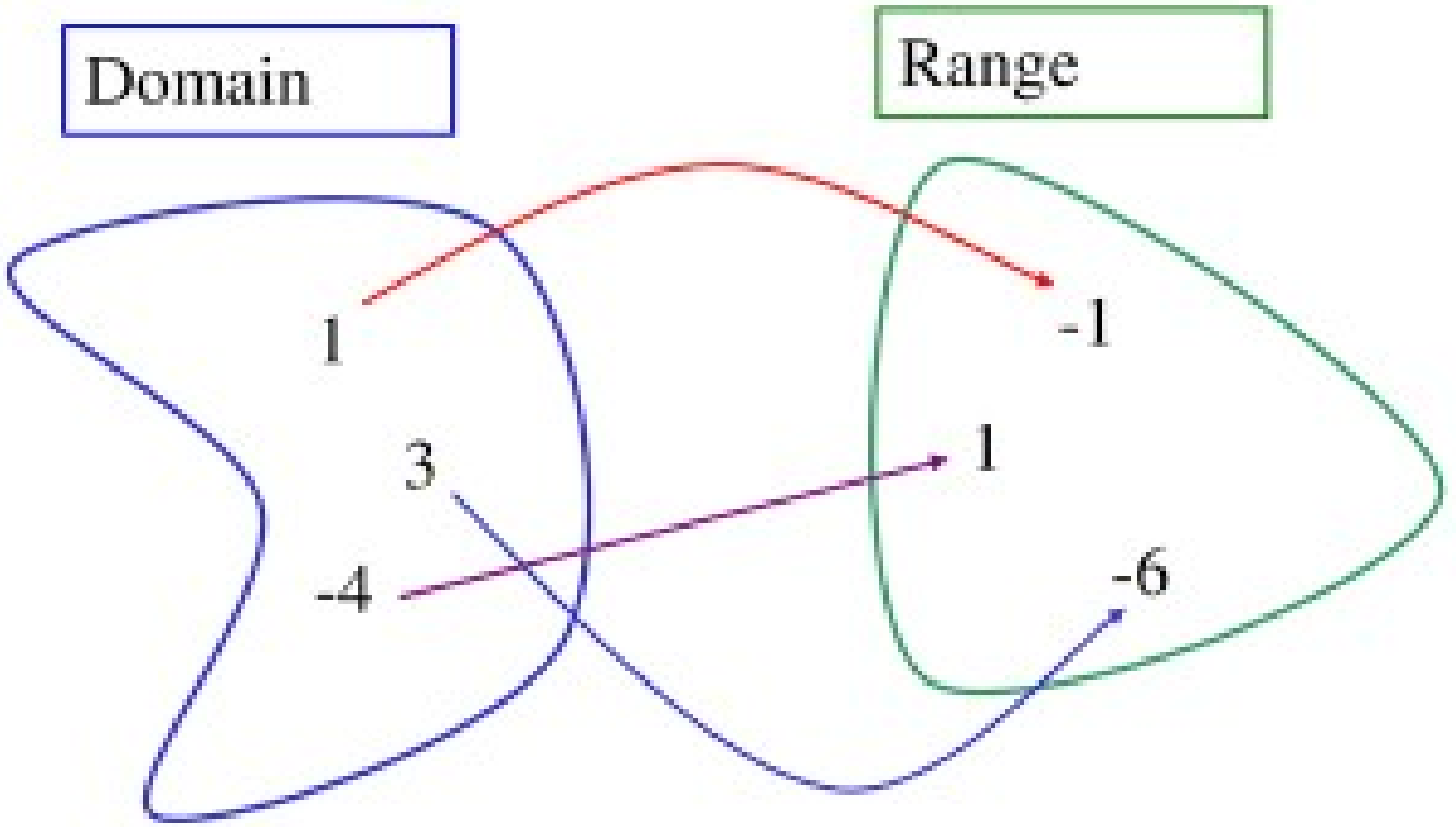
A **function** is a rule that assigns to each element from one set exactly one element from another set.



General form: (Read as “ is a function of ”)

Ex:

Here is the independent variable and is the dependent variable.



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Function Notation

$y = 3x^2 + 2$ is a *function*, with values of x as the domain and values of y as the range.

We write $f(x)$ in place of y.

This is read "f of x."

So $f(x) = 3x^2 + 2$

NOTE: It is not f times x

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Evaluation of a function

Ex: Let . Find the following

(a)

(b) (Replace with)s

(c) Find all values of such that

Thus (Quadratic Eqn, does not factor)

(here , and)

Hence or

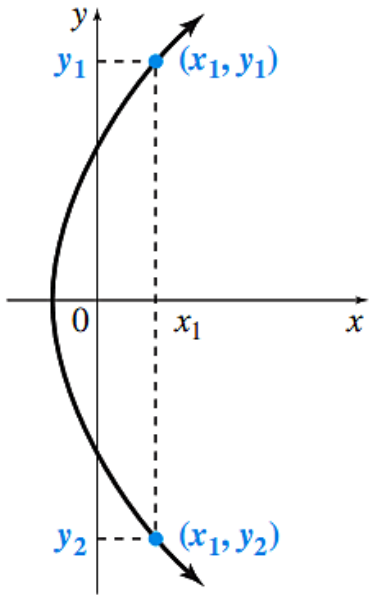
Graph of a Function

Graph: The **graph** of an equation is the set of all points in the plane whose coordinates satisfy the equation.

Note: if a graph is to represent a function, each value of from the domain must lead to exactly one value of . A graph represents a function if and only if every vertical line intersects the graph in no more than one point.

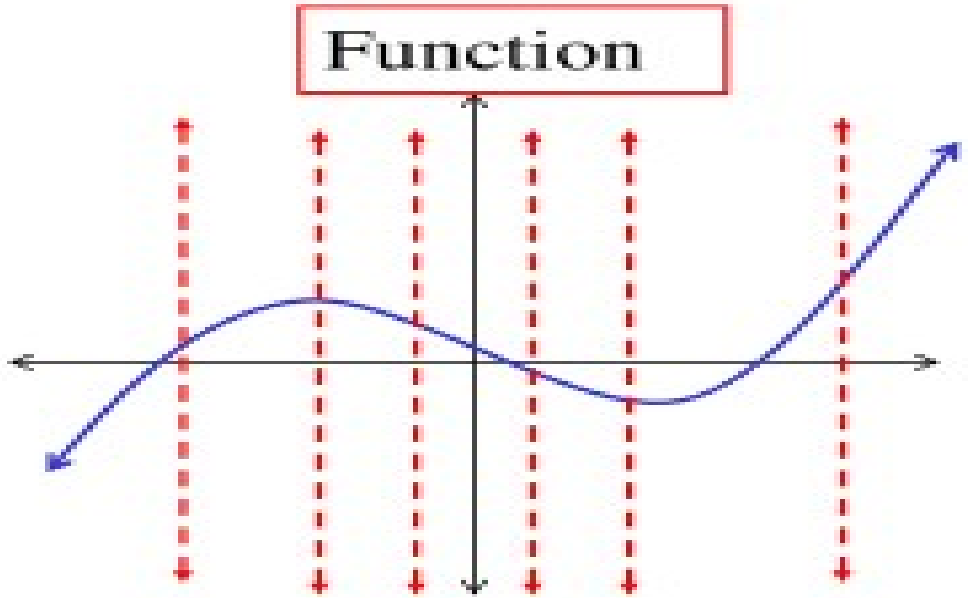
Vertical Line Test

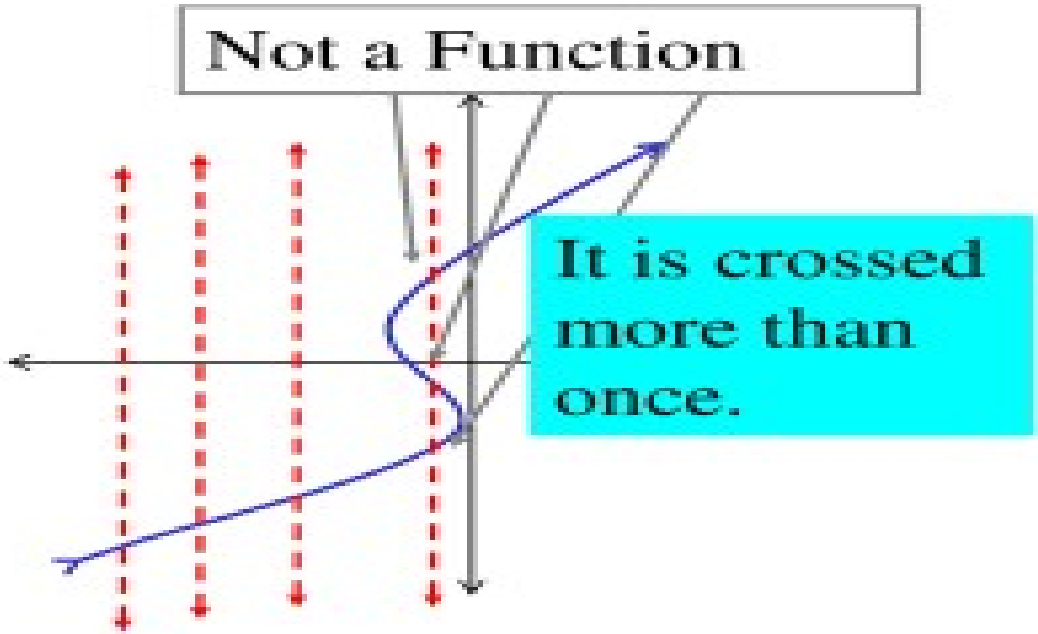
If a vertical line intersects a graph in more than one point, the graph is not the graph of a function.



Graph of a Function

Vertical Line Test: The graph of a function can be crossed at most once by any vertical line.





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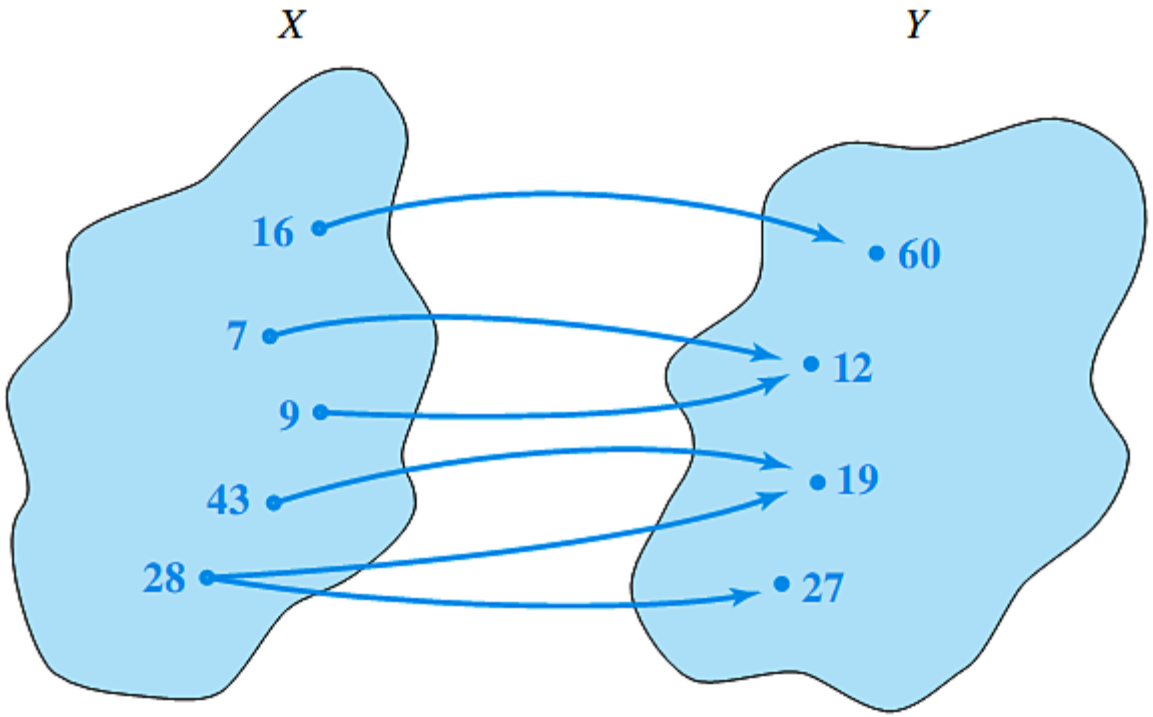
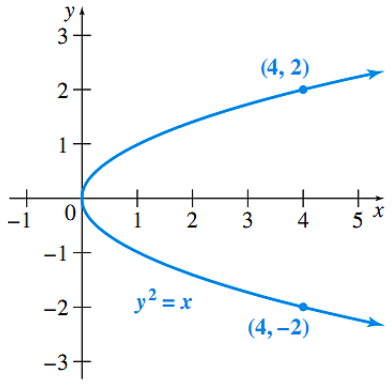
Which of the following are functions?



Soln: The above figure shows that an -value of 28 corresponds to *two* -values, 19 and 27. In a function each must correspond to exactly one , so this correspondence is not a function.

Determine whether each equation or graph represent a function.

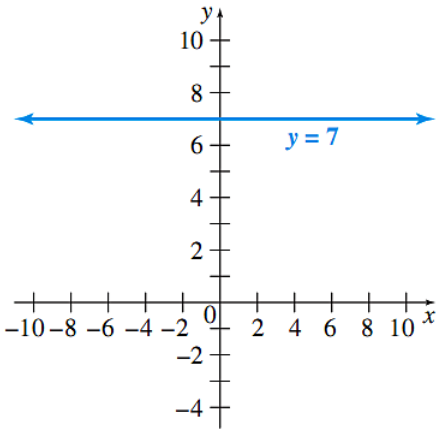
(a) Thus since one value of leads to two values of , is not a function. Also by Vertical line test. we see that it is not a function.



(b)

(b) It is a function because each value of the ns exactly one element 7. The domain is and the ran

By Vertical line test we see that it is a function as i the graph in at most one point.

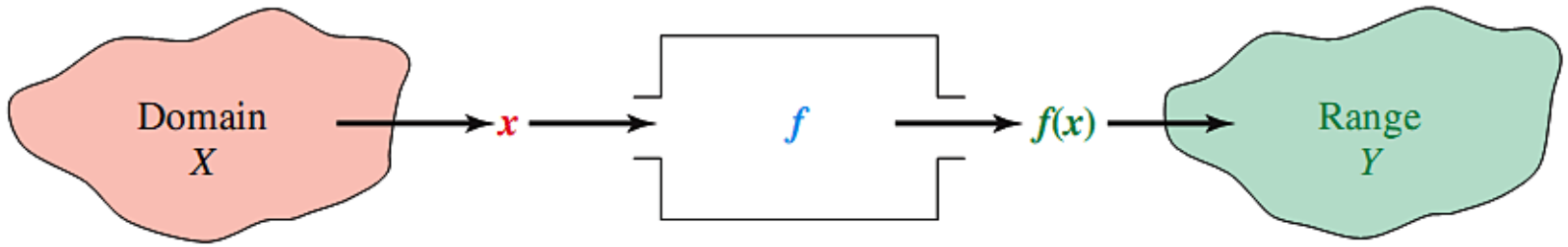


(c)

Domain and Range:

Domain and Range

The set of all possible values of the independent variable in a function is called the **domain** of the function, and the resulting set of possible values of the dependent variable is called the **range**.



- If , then the set of all values for which the is defined is called the domain of and the corresponding set of values when varies over the domain is called the range of
- or To determine the range solve for , then the set of all values for which the is defined is called the range of

How to find the Domain and Range from Graphs

Note: 1)The domain and range of a function can be identified by using graphs. The domain of a graph consists of all input values on the -axis and the range consists of all output va

2) Domain and range are always written fr

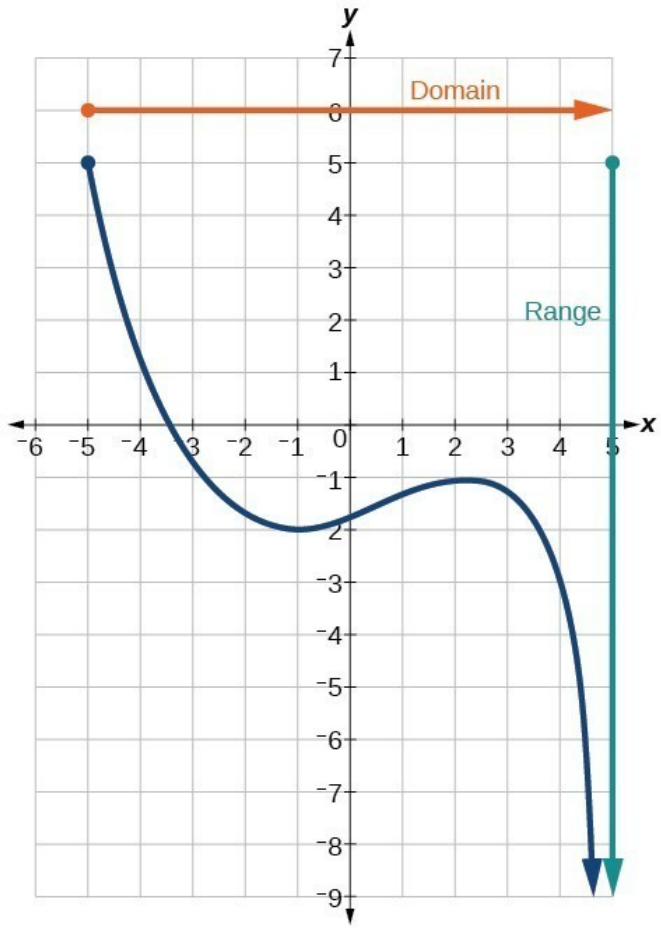
To larger values, or from left to right for domain.

3) Solid circle indicates the number -5 is th

Solution, open circles are used otherwise.

Domain: Domain :

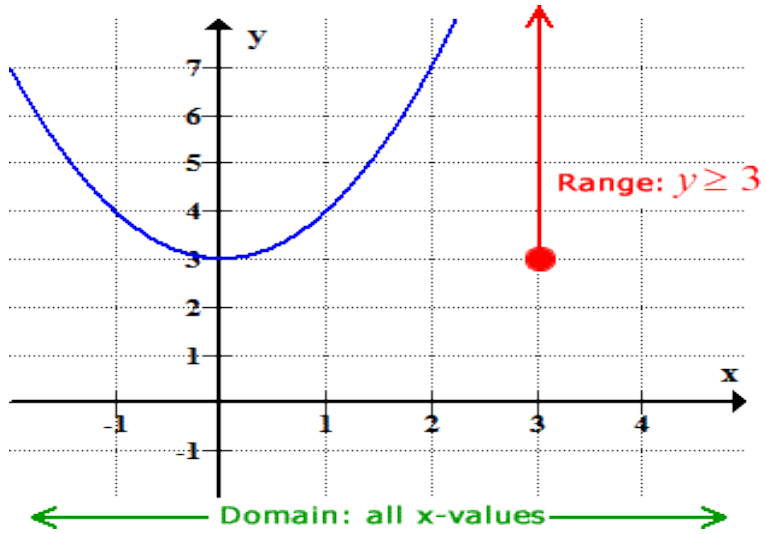
Range:



Ex: Find the domain and range for the following functions

(a)

Soln:



Domain:

There is no -value that can make the function undefined. So the domain is the set of all real numbers, i.e. . In interval notation we can write .

Range: (solve for)

From the graph we can see that the parabola has a low point at and it can go as high as it wants. Thus the range is , i.e. . In other words the range is

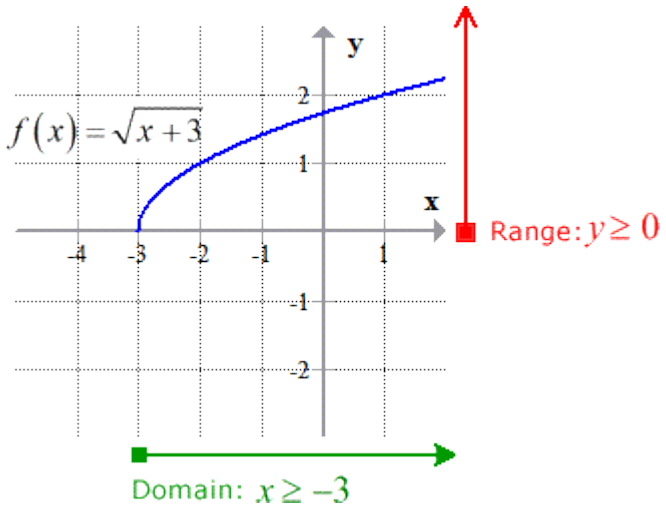
H.W. Show that the domain and range of are as follows:

Domain: , Range:

Cont...

(b) Ex:

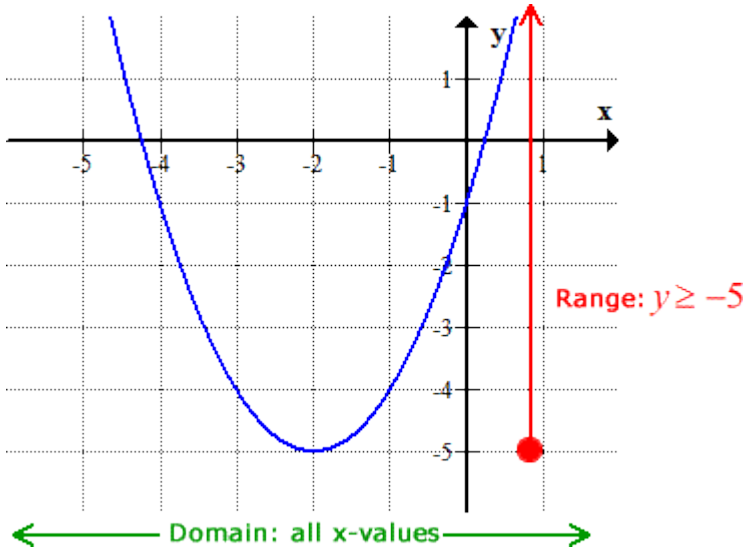
Soln: For to be a real number or . So the domain is given by , i.e. . The range is , i.e. .



Cont…

(c)

Soln:



The domain consists of all values of , , i.e. .

Completing the square we get

The parabola opens upward and the vertex must be a minimum. The graph shows that the parabola has a minimum value at and it can go as high as it wants.

Thus the range is , i.e. . In other words the range is

Cont...

(d)

Soln:

,

,

First solve ,

The **domain** is , i.e. .

Thus the **range** is , i.e. . In other words the range is

Cont…

(e)

Soln:

The **domain** is , any value can be chosen for x.

will either be zero or positive

Thus the **range** is , i.e. . In other words the range is

(f)

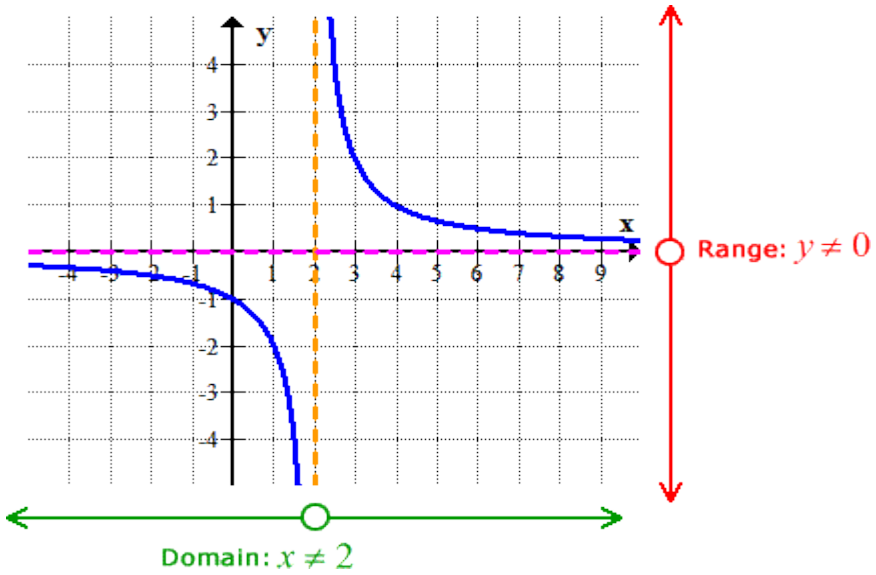
Soln: .

Domain:

Since the denominator cannot be zero, so . The domain can take all value except for 2 So the domain is , i.e.

Range:

Solve for



Because the denominator can never be zero, from above equation

The range can take all value except for 0, so the range is , i.e.

(g) .

Soln:

Domain:

the denominator cannot be zero, **and , i.e** , and

and

In the numerator**, i.e**

so the domain is [4,5

Range

Interval [4,5), as x approaches 5 from the left, the denominator is negative, i.e

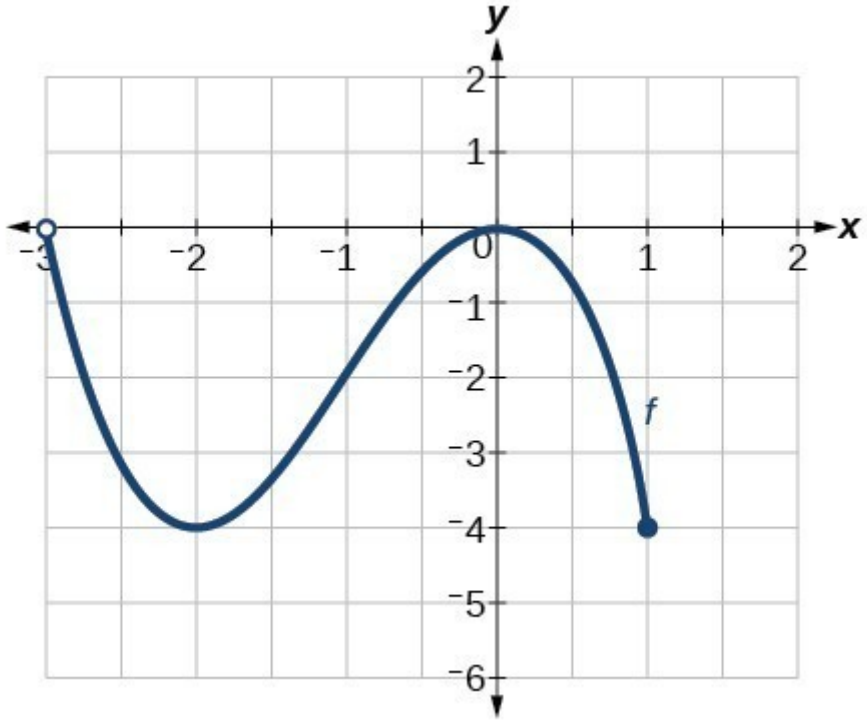
Interval , as x approaches 5 from the right, the denominator is positive i.e

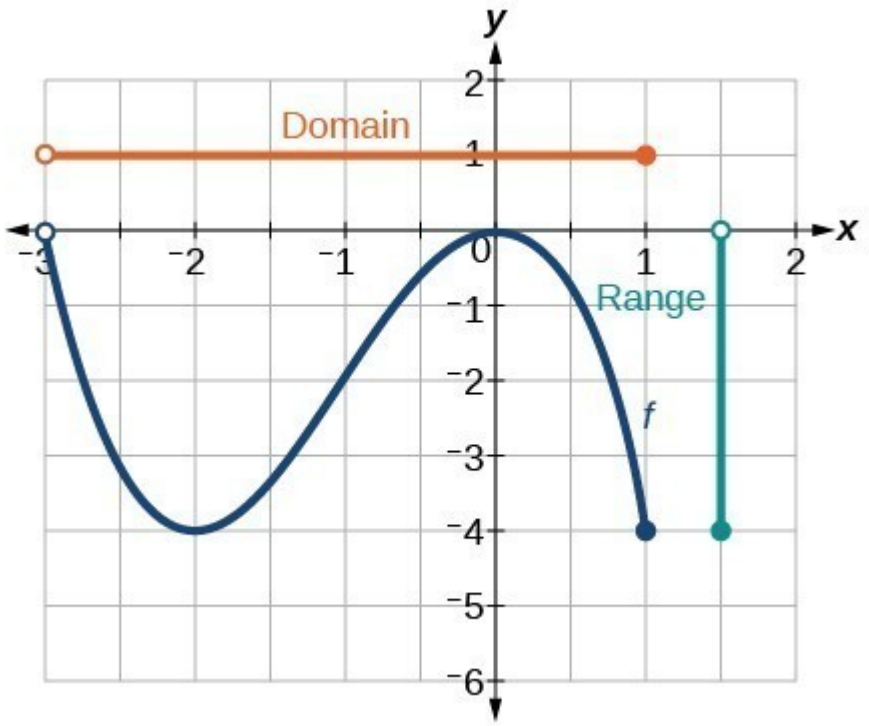
So the range is

.

Find the domain and range from graphs

(g)





Soln : Domain :

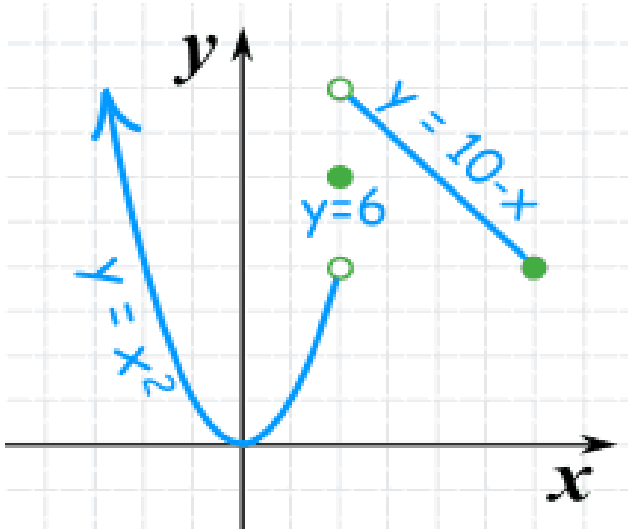
Range:

Piecewise Defined Functions:

A **piecewise-defined function** (also called a **piecewise function**) is a function defined by multiple sub-functions, each sub-function applying to a certain interval of the main function's domain, a sub-domain. For example

Graph of : Solid dot means “including” and open dot means “not including”

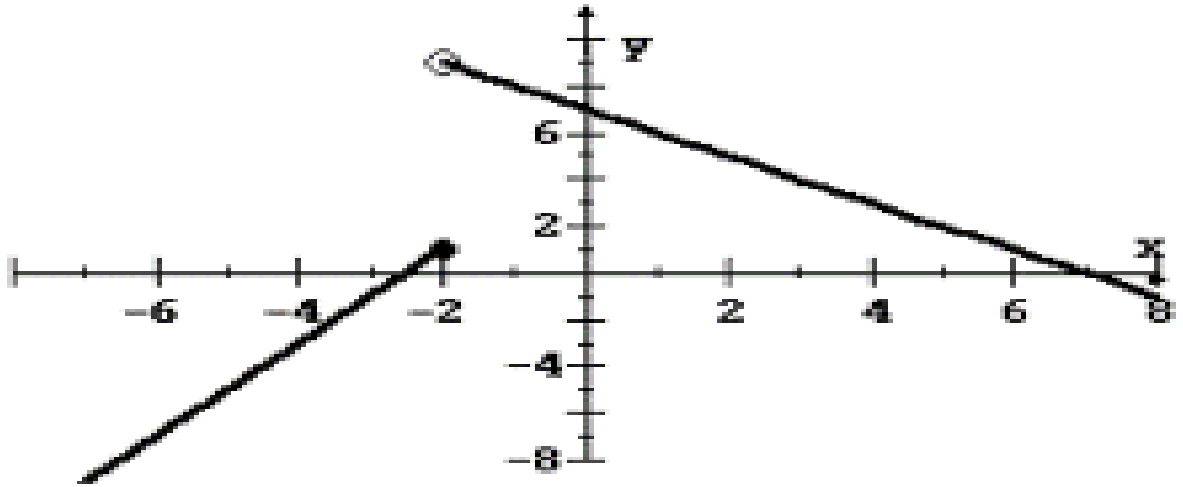
Domain of :



Example

Sketch the graph of the function:

$$f(x) = \begin{cases} 2x + 5 & x \leq -2 \\ -x + 7 & x > -2 \end{cases}.$$



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Examples: Some famous piecewise function

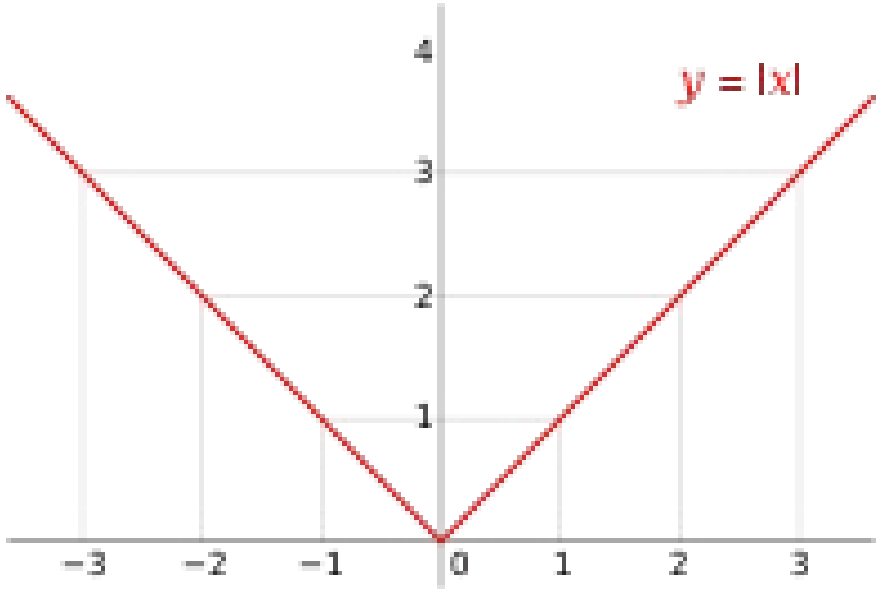
a) Absolute Value Function:

Evaluate

, ,

,

5,

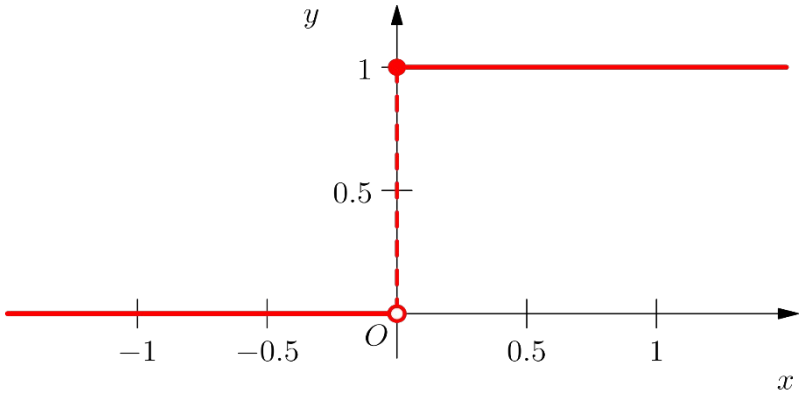


Absolute Value

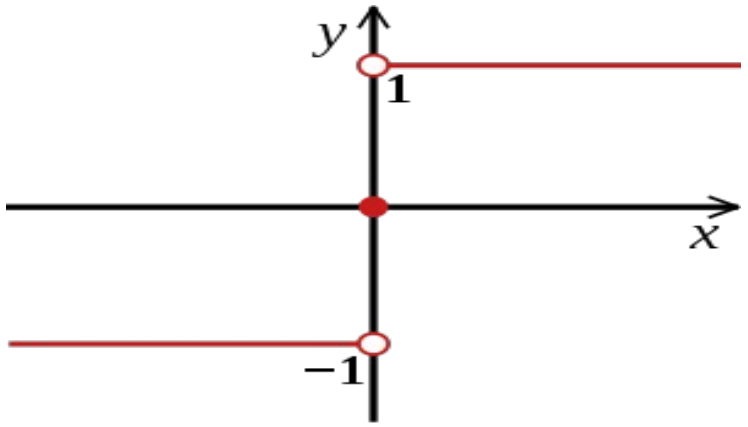
1. The absolute value function $y = f(x) = |x|$ is defined by
$$y = f(x) = \begin{cases} x, & x \geqslant 0 \\ -x, & x < 0 \end{cases}$$
2. Properties of absolute value
3. $|-a| = |a|$
4. $|ab| = |a||b|$
5. $|a/b| = |a|/|b|, b \neq 0$
6. $|a+b| \leq |a| + |b|$
7. $\sqrt{x^2} = |x|$

Cont…

b) **Heaviside Step Function:** Also known as Heaviside unit step function



c) **Sign Function:**



Even Function and Odd Function

A function is an **even function** if .

Ex: is an even function because

The graph is symmetrical about -axis. (Graph (a) below)

A function is an **odd function** if

Ex: is an odd function because

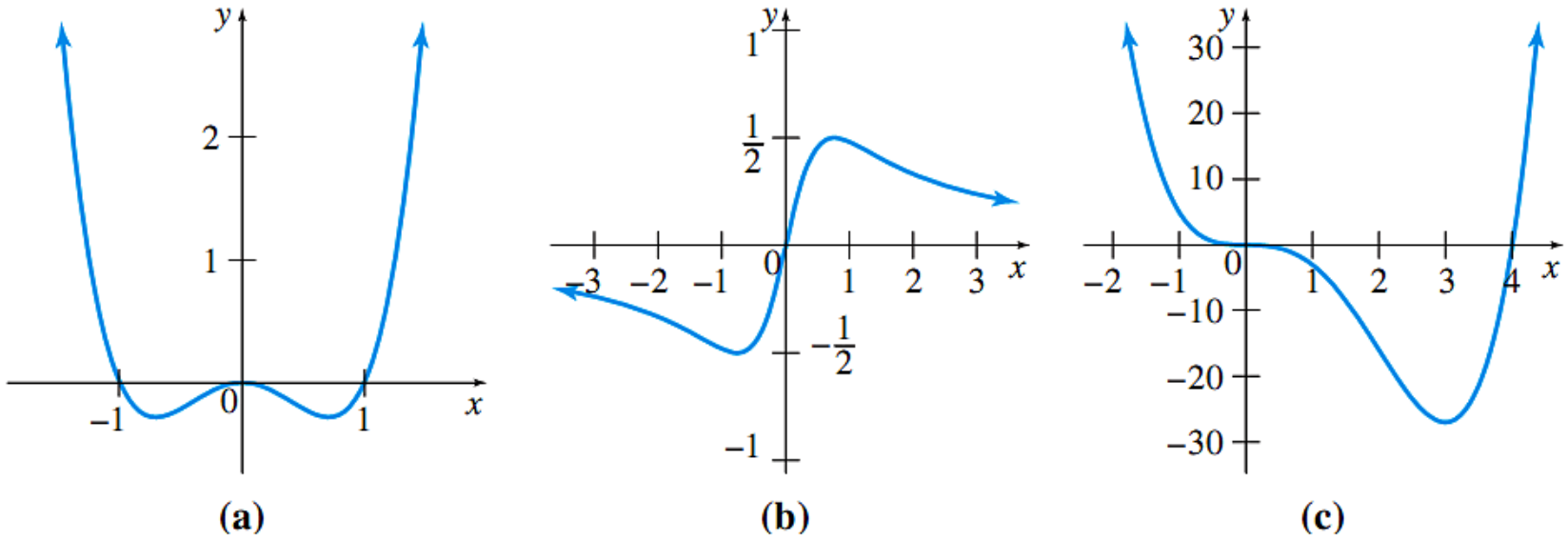
The graph is symmetrical about the origin. (Graph (b) below)

Cont...

Ex: is neither even nor odd, because

or

(Graph (c) below)



Types of Functions:

- Polynomial Functions :

is a non negative integer, each is a constant

e.g .

9 (Constant function)

(Linear function)

(Quadratic function)

(Cubic function)

` (Quatic function)

Power function (r is any real number) e.g

- Rational Function : , and , are polynomial) e.g
- Exponential Function: e.g or
- Logarithmic Function: e.g or
- Trigonometric Functions: e.g

1.2 Combining Functions, Shifting and Scaling

Sùms, Difference, Products and Quotients

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$

Domain:

Domain of f intersected with the domain of g.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Domain:

Domain of f intersected with the domain of g with the exclusion of all values of x, such that $g(x) = 0$.

Given $f(x) = 3x - 1$ and $g(x) = x^2 - 2x - 3$

$$(f+g)(x) = 3x - 1 + x^2 - 2x - 3$$

$$= x^2 + x - 4$$

$$(f-g)(x) = 3x - 1 - (x^2 - 2x - 3)$$

$$= -x^2 + 5x + 2$$

$$(fg)(x) = (3x-1)(x^2 - 2x - 3)$$

$$= 3x^3 - 7x^2 - 7x + 3$$

$$\left(\frac{f}{g}\right)(x) = \frac{3x-1}{x^2 - 2x - 3}, \quad (x^2 - 2x - 3 \neq 0)$$

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Composing Functions

If and are functions, the function composed with) is defined by

and called the composition of and The domain of consist of the numbers in the domain for which lies in the domain of

Ex. Given $f(x)=\frac{2}{3x-5}$, $g(x)=x^3-3$ find $(f\circ g)(x)$ and $(g\circ f)(1)$.

$$(f\circ g)(x)=f(g(x))=f\left(x^3-3\right)$$

$$=\frac{2}{3\left(x^3-3\right)-5}=\frac{2}{3x^3-14}$$

$$(g\circ f)(1)=g(f(1))=g(-1)=-4$$

Polynomial Functions, Translation and Reflection

Quadratic Function

A **quadratic function** is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real numbers, with $a \neq 0$.

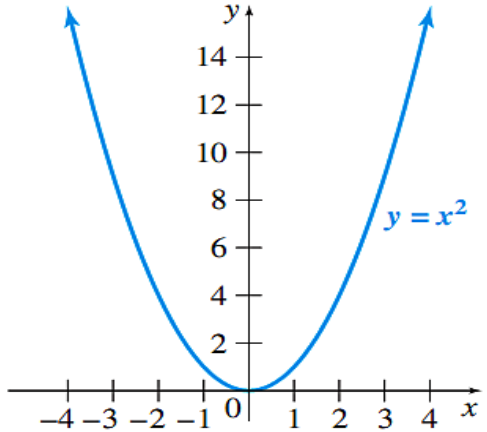
Every quadratic function has a parabola as its graph.
Simplest quadratic function: (

The lowest (or highest) point of a parabola i

Vertex of the parabola. The graph of a qua

Function is symmetric with respect to the ve

Line through the vertex , i.e. the **axis** of the



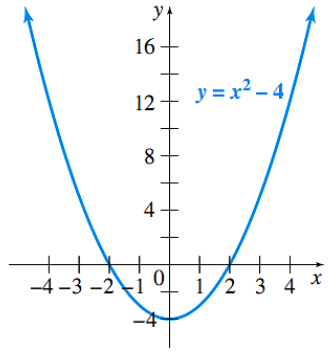
Note: 0, Parabola opens upward and

, Parabola opens downward

Graphing a Quadratic Function

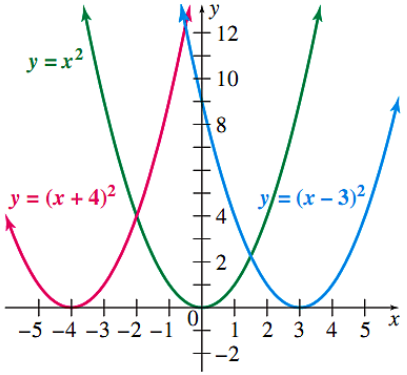
Ex: Graph

The graph has the same shape as that of but is 4 units lower.



Ex: Graph of for =*3*, *0* and *-4*

For the graph of shifted 3 units to the right, similarly for , the graph shifts to the left 4 units. This is known as **horizontal translation** of the function.



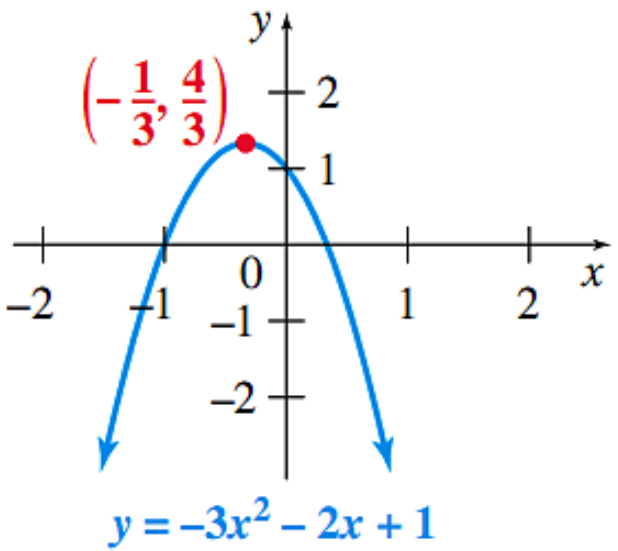
Graphing a Quadratic Function

Graph of the Quadratic Function

The graph of the quadratic function $f(x) = ax^2 + bx + c$ has its vertex at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right).$$

The graph opens upward if $a > 0$ and downward if $a < 0$.



Ex: Graph

Soln:

(Completing the square)

It has the form with and , the graph of translated 1/3 unit to the left and 4/3 units upward, vertex at (-1/3, 4/3). Since (negative), the graph will open downward.

Note: To identify the translation and any vertical reflection, we can rewrite the quadratic equation in the form . In this form, the vertex is .

Translations of Functions

- Horizontal shift

shift the graph of “” units to the right

shift the graph of “” units to the left

Vertical shift

shift the graph of “” units up

shift the graph of “” units down

- To obtain the graph of from the graph of , first move the graph of horizontally by “” units to the right or left and then move this graph by “” units vertically up or down.

Ex: To obtain the graph of ,

Reflections of Functions

- To obtain the graph of from the graph of , reflect the graph of about the -axis.

- To obtain the graph of from the graph of , reflect the graph of about the -axis

Translations of Functions

Translations and Reflections of Functions

Let f be any function, and let h and k be positive constants (Figure 21).

The graph of $y = f(x) + k$ is the graph of $y = f(x)$ translated upward by an amount k (Figure 22).

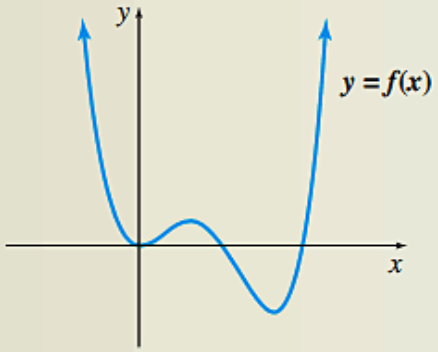


FIGURE 21

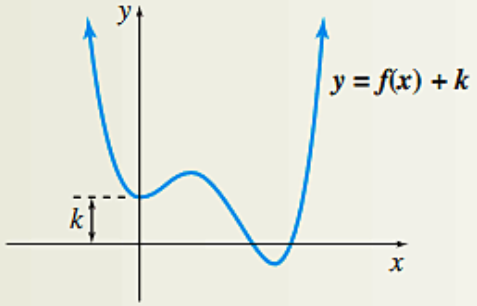


FIGURE 22

The graph of $y = f(x) - k$ is the graph of $y = f(x)$ translated downward by an amount k (Figure 23).

The graph of $y = f(x - h)$ is the graph of $y = f(x)$ translated to the right by an amount h (Figure 24).

The graph of $y = f(x + h)$ is the graph of $y = f(x)$ translated to the left by an amount h (Figure 25).

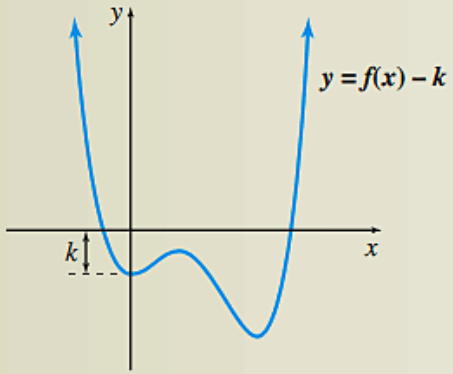


FIGURE 23

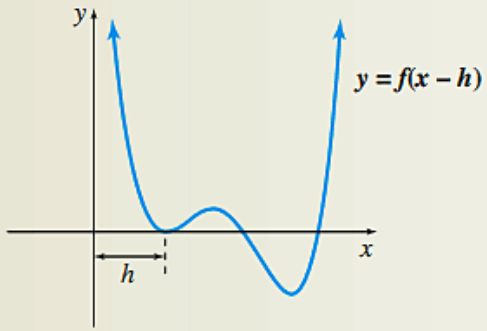


FIGURE 24

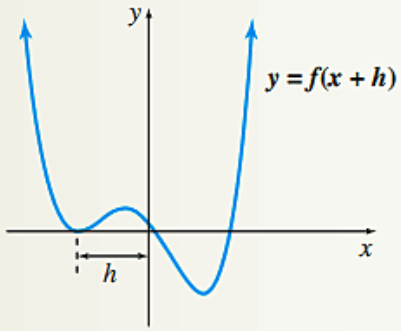


FIGURE 25

Reflections of Functions

Translations and Reflections of Functions

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected vertically across the x-axis, that is, turned upside down (Figure 26).
The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected horizontally across the y-axis, that is, its mirror image (Figure 27).

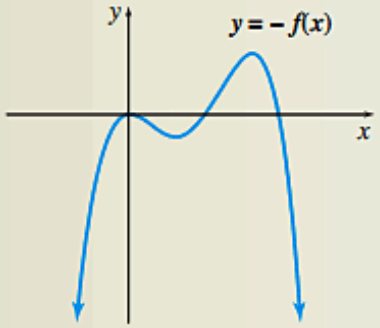


FIGURE 26

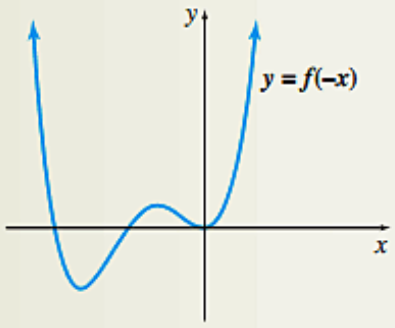


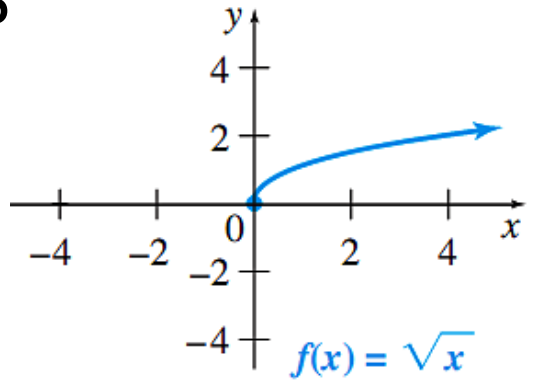
FIGURE 27

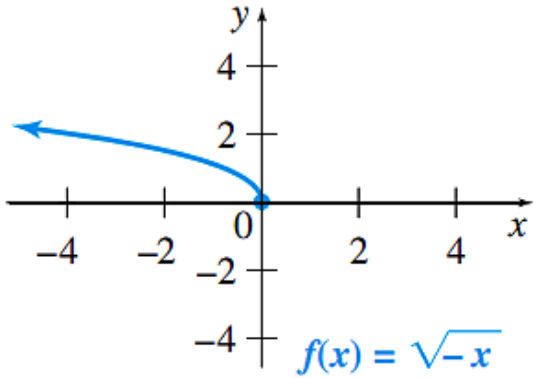
There is another type of reflection, known as horizontal reflection. Multiplying or by a constant , to get or does not change the general appearance of the graph, except to compress or stretch it.

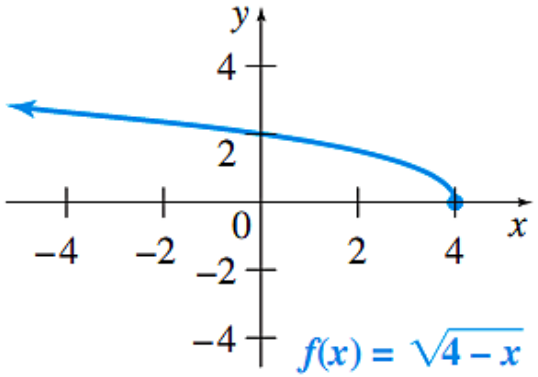
Translations and Reflection of a Graph

Ex: Graph

Solu







Translations and Reflection of a Graph Cont...

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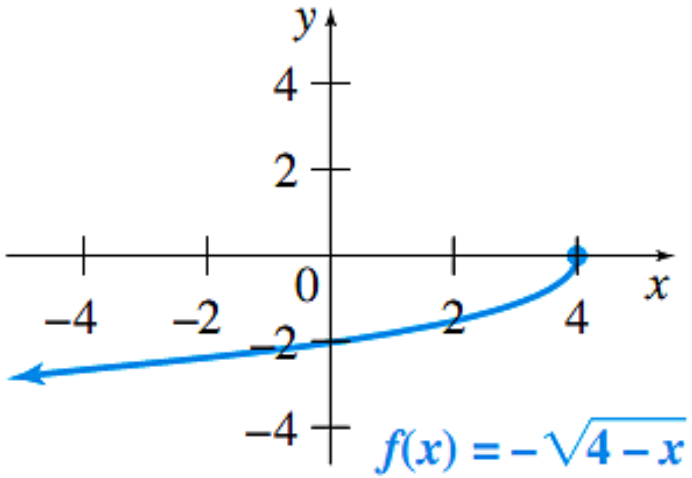
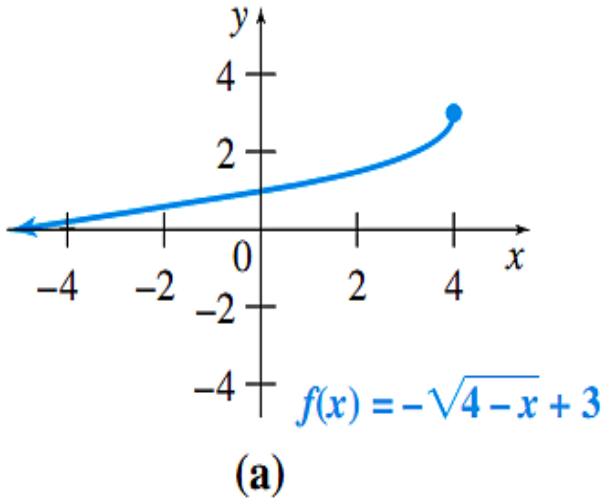
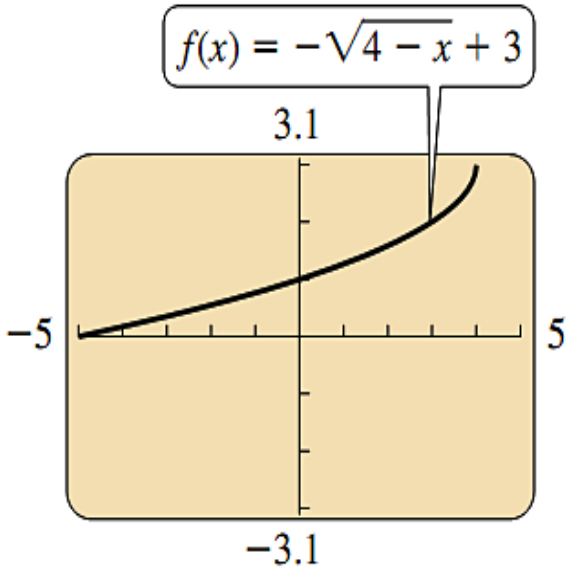




(a)

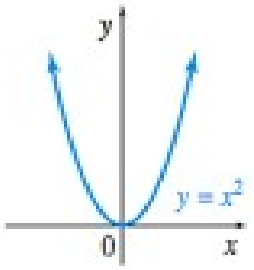


(b)

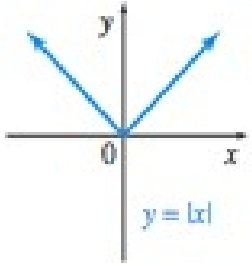
Basic graphs

Graph of Basic Functions

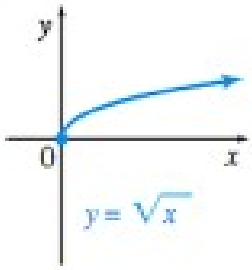
Quadratic



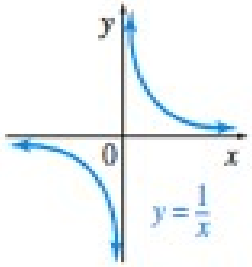
Absolute Value



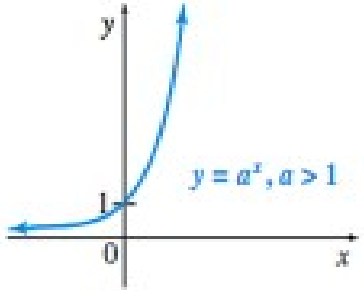
Square Root



Rational



Exponential



Logarithmic